Aims of session

- Last week
- Deterministic
  - Propositional logic
  - Predicate logic

- This week (Basis of this section Johnson and Picton 1995)
- Non-monotonic logic
- Non-deterministic
  - Bayesian
  - Fuzzy Logic
Non-Monotonic Logic

- Something is Monotonic if the number of conclusions that can be drawn from a set of propositions does not DECREASE if new propositions are discovered.
Non-Monotonic Logic

- But can be get something where this is not TRUE.
Yes using an example (Johnson and Picton, 1995, pg 190)

- X: power to robot
- Y: safety devices in place
- P: robot operates.

\[ T(P) = T(X \text{ AND } Y) \]
Later a new proposition is added:
- Z: adequate lubricant

\[ T(P) = T(X \text{ AND } Y \text{ AND } Z) \]

So it is now possible for \( T(P) \) to be FALSE now even if \( T(X) \) and \( T(Y) \) are both TRUE.

So a new proposition has altered a previous conclusion, this should not happen with monotonic logic.
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So a new proposition has altered a previous conclusion, this should not happen with monotonic logic.
Bayes Rule

- From probability theory you can get the probability of an event occurring.

- What can be done with this though?

- We can try to determine the likelihood of an being TRUE given some evidence which itself has a certain probability of being true.
Bayes Rule

- We can try to determine the likelihood of an being TRUE given some evidence which itself has a certain probability of being true.
Bayes Rule

\[ p(A|B) = \frac{p(B|A)p(A)}{p(B)} \]

- Where \( p(A|B) \) is the probability of \( A \) occurring given \( B \) has happened.
- \( P(B|A) \) probability of \( B \) happening, given than \( A \) has happened.
- \( A \) and \( B \) are two independent events.
Example

- A sensor detects a high temperature, what is the probability that this is due to a leak in cooling system?
- We need to some statistical information to use this tool.
Example

- Such as:
  - Total working life (time the statistics have been collected over): 10000 hours.
  - No. of hours the temperature has been high: 42 hours.
  - No. of hours that the system has had a leak in cooling systems: 32 hours.
- P(A)
  - probability of a leak = \( \frac{32}{10000} = 0.0032 \)
- P(B)
  - Probability of a high temp
    - \( \frac{42}{10000} = 0.0042 \)
- Probability of system getting hot when there is a leak in the system is definite so therefore P(B|A) = 1.
\[ P(A | B) = \frac{P(B | A) \cdot P(A)}{P(B)} \]

\[
= \frac{0.0032}{0.0042}
\]

\[= 0.762 \]
What does it mean?

- We can 76% confident that the cooling system is the cause of the high temperature.

- So we can use this as part of a decision making system.
Probability and logic
Introduction to Fuzzy Logic

- Lofti Zadeh (1965) proposed Possibilistic Logic which became Fuzzy-Logic.
- Allows us to combine weighting factors with propositions.
- $0 \leq T(X) \leq 1$
# Boolean v Fuzzy

<table>
<thead>
<tr>
<th>Boolean</th>
<th>Fuzzy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T(X \land Y)$</td>
<td>$\min(T(X), T(Y))$</td>
</tr>
<tr>
<td>$T(X \lor Y)$</td>
<td>$\max(T(X), T(Y))$</td>
</tr>
<tr>
<td>$T(\neg X)$</td>
<td>$(1 - T(X))$</td>
</tr>
<tr>
<td>$T(X \rightarrow Y)$</td>
<td>$\max((1 - T(X), T(Y))$</td>
</tr>
</tbody>
</table>

Where $X$ and $Y$ are propositions.

Any Boolean expression can be converted to a fuzzy expression.
Membership functions

- A fuzzy set is a set whose membership function takes values between 0 and 1.
- Example: Cold, Warm and Hot describe temperature we could define thresholds $T_1$ and $T_2$.
- Starting at low temperature as the temperature rises to $T_1$ the temperature becomes Warm. As the temperature rises to $T_2$ the temperature becomes Hot.
What is the problem?

- Is there really a crisp change between the definitions?
Answer

- Change the shape of the membership function so it not so crisp.
- Common one is a triangular functions that have some overlap.
- At some temperatures it is possible to be a member of two different sets.
Using the example from Johnson and Picton (1995)
At 8 degrees it is a member of both COLD (0.7) and WARM (0.3) sets.

These are NOT necessarily probabilities, they are not so rigorously defined.
Defuzzication

- To calculate final setting need defuzzication rules, this often based around the ‘centre of gravity’ of shaded area.

- Why do we need this?
- So back to the temperature measures the fuzzy membership can be combined using MIN, MAX and (1-T(X)) operations so IF-THEN can be used.
  - IF (temperature is COLD) THEN (heating on HIGH)
  - IF (temperature is WARM) THEN (heating on LOW)

- So first rule heating is turned on to HIGH with a membership of 0.7. Second rule heating is turned on to LOW.

- So membership can be represented by the heating membership,
Heater membership

- The centre of gravity is the point where the area to left of the point equals the area to the right.
Centre of Gravity

\[
\text{cofg} = \frac{\sum_{i=1}^{N} \text{cofg}_i \cdot \text{area}_i \cdot \text{under}_i \cdot \text{curve}_i}{\sum_{i=1}^{N} \text{area}_i \cdot \text{under}_i \cdot \text{curve}_i}
\]
Not inverse

- Defuzzication is not truly the inverse of fuzzification.

- If you defuzzify fuzzy data you will often get distortion in the resulting values.
References