**Artificial Intelligence Techniques**

Knowledge Processing 3

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**Aims of session**

To understand
- Fuzzy Logic
- Defuzzification

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**Introduction**

- Lofti Zadeh (1965) proposed Possibilistic Logic which became Fuzzy-Logic.
- Allows us to combine weighting factors with propositions.
- $0 \leq T(X) \leq 1$
### Boolean v Fuzzy

<table>
<thead>
<tr>
<th>Boolean</th>
<th>Fuzzy</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(X ^ Y)</td>
<td>MIN(T(X),T(Y))</td>
</tr>
<tr>
<td>T(XvY)</td>
<td>MAX(T(X),T(Y))</td>
</tr>
<tr>
<td>T(¬X)</td>
<td>(1−T(X))</td>
</tr>
<tr>
<td>T(X→Y)</td>
<td>MAX((1−T(X)),T(Y))</td>
</tr>
</tbody>
</table>

Where X and Y are propositions
Any Boolean expression can be converted to a fuzzy expression.

### Membership functions

- A fuzzy set is a set whose membership function takes values between 0 and 1.
- Example: Cold, Warm and Hot describe temperature we could define thresholds T1 and T2.
- Starting at low temperature as the temperature rises to T1 the temperature becomes Warm. As the temperature rises to T2 the temperature becomes Hot.

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>MIN(X,Y)</th>
<th>MAX(X,Y)</th>
<th>(1−X)</th>
<th>MAX((1−X),Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
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So when truth values are 0 and 1 Fuzzy=Boolean
Other rules such as De-Morgan’s Laws still apply
What is the problem?
- Is there really a crisp change between the definitions?

Answer
- Change the shape of the membership function so it not so crisp.
- Common one is a triangular functions that have some overlap.
- At some temperatures it is possible to be a member of two different sets.

Using the example from Johnson and Picton (1995)
At 8 degrees it is a member of both COLD (0.7) and WARM (0.3) sets. These are NOT necessarily probabilities, they are not so rigorously defined.

Defuzzication

To calculate final setting need defuzzication rules, this often based around the ‘centre of gravity’ of shaded area.

Why do we need this?

So back to the temperature measures the fuzzy membership can be combined using MIN, MAX and (1-T(X)) operations so IF-THEN can be used.

- IF (temperature is COLD) THEN (heating on HIGH)
- IF (temperature is WARM) THEN (heating on LOW)

So first rule heating is turned on to HIGH with a membership of 0.7. Second rule heating is turned on to LOW.

So membership can be represented by the heating membership,
Heater membership

- Centre of gravity is point where area to left of the point = area to the right.

Centre of Gravity

\[ \text{cofg} = \frac{\sum \text{cofg} \cdot \text{area under curve}_i}{\sum \text{area under curve}_i} \]

Paradoxes and Fuzzy Sets

- A useful feature is they can be built on the basis of minimal information and fine-tuned to be more consistent afterwards by observation.
- Problem is (Hopgood’s Paradox) is that it is possible that ‘weak’ information can result in strong information on defuzzication.
Defuzzication is not truly the inverse of fuzzification.

If you defuzzify fuzzy data you will often get distortion in the resulting values.

References